2022

MATHEMATICS

(Honours)

Paper Code : VIII - A & B

(New Syllabus)

Full Marks : 60 Time : Three Hours

Paper Code : VIII - A

(Marks : 10)

Choose the correct answer.

Each question carries 2 Marks.

1. The rank of the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (y, 0, z)$ is —
   (A) 3
   (B) 2
   (C) 1
   (D) 0

2. Which of the following statement is correct ?
   (A) $(S_3, \circ)$ and $(\mathbb{Z}_6, +)$ are isomorphic
   (B) $(\mathbb{R}, +)$ and $(\mathbb{R}^+, \bullet)$ are isomorphic
   (C) $(\mathbb{Q}, +)$ and $(\mathbb{Z}, +)$ are isomorphic
   (D) None of the above

3. If A_{ij} is a skew-symmetric tensor, then $\left(\delta^i_j \delta^k_l + \delta^i_l \delta^k_j\right) A_{ik}$ is equal to —
   (A) 0
   (B) 1
   (C) 2
   (D) None of the above

4. In a Boolean algebra B, for all $a, b \in B$, the value of $\left[\left(a'+b'\right).\left(a+b'\right)\right]'$ is —

(A) $a+b$

(B) $a+b'$

(C) $a'+b$

(D) 1

5. The value of $\boldsymbol{L}^{-1}\left\{\frac{-2s+6}{s^2+4}\right\}$ is —

(A) $-2\sin(2t)+3\cos(2t)$

(B) $-2\sin(t)+3\cos(t)$

(C) $-2\cos(t)+3\sin(t)$

(D) $-2\cos(2t)+3\sin(2t)$

Paper Code : VIII - B

(Marks : 50)

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *two* questions : 4×2=8

(a) Let $T : P(\mathbb{R}) \to P(\mathbb{R})$ be a mapping defined by

$$T\left(f(x)\right) = \int_0^x f(u)\,du$$

for all $f(x) \in P(\mathbb{R})$, where $P(\mathbb{R})$ stands for the set of all polynomials with real coefficients. Prove that T is linear and one-to-one, but not onto. 2+1+1

(b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear mapping defined by

$$T(x, y, z) = (x - y, x + 2y, y + 3z)$$

for all $(x, y, z) \in \mathbb{R}^3$. Show that T is non-singular and determine T^{-1}. 2+2

(c) If the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ be such that $T(1, 0) = (2, 3, 1)$, $T(1, 1) = (3, 0, 2)$. Then find $T(a, b), (a, b) \in \mathbb{R}^2$. Also find $Ker(T)$. 3+1

2. Answer any *two* questions : 3×2=6

(a) Let G be a non-commutative group with centre $Z(G)$. Prove that $G/Z(G)$ is non-cyclic. 3

(b) Let $\phi(G, \circ) \to (H, *)$ be a homomorphism. Prove that, ϕ is injective iff $Ker\,\phi = \{e\}$, where e being the identity element of G. 3

(c) Find the permutation group isomorphic to the Klein's 4-group. 3

3. Answer any *two* questions : 3×2=6

(a) Prove that there does not exist a Boolean algebra containing only three elements. 3

(b) A Boolean function f is defined by

$$f(x, y, z) = xy + yz + zx$$

Determine the conjunctive normal form of $f(x, y, z)$. 3

(c) Design a simple circuit connecting two wall switches and a light bulb in such a way that either switch can be used to control the light independently. 3

4. Answer any *three* questions : 5×3=15

(a) If $L\{F(t)\} = f(p)$ and $G(t) = F(t-a)$, $t \geq a$ show that $L\{G(t)\} = \exp(-ap)\, L\{F(t)\}$. Hence evaluate the Laplace transform of

$$F(t)\begin{cases} \sin\left(t - \pi/3\right) & \text{if } t \geq \pi/3 \\ 0 & \text{if } t < \pi/3 \end{cases}$$ 3+2

(b) Evaluate $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t}\, dt$. 5

(c) Using Laplace transformation, solve

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 2 - e^{2t},$$

given $y(0) = 1$ and $\frac{dy}{dt}(0) = 0$. 5

(d) Determine the series solution of the differential equation

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

near the ordinary point x=0. 5

(e) Find $L^{-1}\left\{\frac{1}{S}\log\frac{S+2}{S+1}\right\}$. 5

5. Answer any *three* questions : 5×3=15

(a) If the relation $b^{ij}u_i u_j = 0$ holds for any arbitrary covariant vector u_i, prove that $b^{ij} + b^{ji} = 0\cdot$ 5

(b) Assume that $A^i B_i$ is an invariant for all contravariant vector A^i. Then show that B_i is a covariant vector. 5

(c) Let us consider the three-dimensional Euclidean space $\mathbb{E}^3$. Determine the metric tensor in cylindrical polar coordinates. 5

(d) Show that the covariant derivative of δ_{ij} and g_{ij} are zero. 5

(e) In Minkowski space-time with line element

$$ds^2 = -\left(dx^1\right)^2 - \left(dx^2\right)^2 - \left(dx^3\right)^2 + c^2\left(dx^4\right)^2,$$

show that the vector $\left(\sqrt{2}, 0, 0, \sqrt{3}/c\right)$ is a unit vector and the vector $\left(-1, -1, 1, \sqrt{3}/c\right)$ is a null vector. 5