2022

MATHEMATICS

(Honours)

Paper Code : VII - A & B

(New Syllabus)

Full Marks : 100 Time : Four Hours

Paper Code : VII - A

(Marks : 20)

Choose the correct answer.

Each question carries 2 Marks.

1. If $\vec{F}$ be a solenoidal vector field then the flux $\int_S \vec{F}.\vec{n}\, ds$ across any closed surface S is —

   (A) 2π

   (B) 0

   (C) -2π

   (D) 4π

2. If S is a closed furface and V is the volume enclosed by S, then the value $\iint_S \vec{r}.\vec{n}.ds$ is —

   (A) $2V$

   (B) $3V$

   (C) V

   (D) $4V$

3. The value of the line integral $\int_C \left(2xy^2dx + 2x^2ydy + dz\right)$ along a path joining the origin and the point (1, 1, 1) is —

   (A) 0

   (B) 2

   (C) 4

   (D) 6

4. The coordinates of c.g. of a circular are of radius 'a' making an angle 2α at the centre are —

(A) $\left(\frac{a \sin \alpha}{\alpha}, 0\right)$

(B) $\left(0, a\frac{\cos \alpha}{\alpha}\right)$

(C) $\left(\frac{a \tan \alpha}{\alpha}, 0\right)$

(D) $\left(0, \frac{a \cot \alpha}{\alpha}\right)$

5. A ball balanced on a vertical rod, it is an example of —

(A) Perfect equilibrium

(B) Neutral equilibrium

(C) Stable equilibrium

(D) Unstable equilibrium

6. A circular area with radius a is immersed in a liquid with its centre at a depth 'a' below the surface. Then the depth of centre of pressure below the free surface is —

(A) $\frac{3}{2}a$

(B) $\frac{3}{4}a$

(C) $\frac{5}{4}a$

(D) $\frac{a}{2}$

7. The moment of inertia of a solid sphere of mass M about a diameter $2a$ is —

(A) $\frac{2}{5}Ma^2$

(B) $\frac{4}{5}Ma^2$

(C) $\frac{1}{5}Ma^2$

(D) $\frac{8}{5}Ma^2$

8. If K be the radius of gyration of a rigid body of mass M about the axis, then kinetic energy of the rigid body rotating with constant angular velocity ω about the axis is —

(A) $MK^2\omega^2$

(B) $\frac{1}{2}MK^2\omega$

(C) $\frac{1}{2}MK^2\omega^2$

(D) $MK^2\omega$

9. The pressure at any given point of a non-moving fluid is called the —

(A) Gauge pressure

(B) Atmospheric pressure

(C) Differential pressure

(D) Hydrostatic pressure

10. A force acting on a body where R is the normal reaction, $\vec{F}$ is the frictional force and μ is the coefficient of friction. Then the body will rolls if —

(A) $\mu > {}^{F}\!/_{R}$

(B) $\mu < \frac{F}{R}$

(C) $\mu = \frac{F}{R}$

(D) $\mu \neq {}^{F}\!/_{R}$

Paper Code : VII - B

(Marks : 80)

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(Marks : 10)

Answer any *two* questions. 5×2=10

1. If $\vec{F}=\left(5x^2+6y\right)\hat{i}-\left(3x+2y^2\right)\hat{j}+2xz^2\hat{k}$, then evaluate $\int_C \vec{F}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path C given by the straight lines from (0, 0, 0) to (1, 0, 0) then to (1, 1, 0) and then to (1, 1, 1). 5

2. State Green's theorem. Show that the area bounded by a simple closed curve C is given by

$$\frac{1}{2}\oint_C x\,dy-y\,dx$$ 2+3

3. Verify Stoke's theorem for the vector function

$$\vec{F}=\left(x^2-y^2\right)\hat{i}+2x\hat{j}$$

around the rectangle bounded by the straight lines

$$x=0,\ x=a,\ y=0,\ y=b.$$ 5

4. Use divergence theorem to evaluate $\iint \vec{F}.\hat{n}\,ds$, where $\vec{F}=4zx\hat{i}-y^2\hat{j}+yz\hat{k}$ and S is the surface of the cube bounded by $x=0, x=1;\ \ y=0, y=1;\ \ z=0, z=1$. 5

Group - B

(Marks : 25)

Answer question no. 5 and any *three* from the rest.

5. Answer any *one* question : 4

   (a) Three forces P, Q, R act along the sides of the triangle formed by the lines

   $$x+y=1,\ y-x=1,\ y=2$$

   Find the equation of the line of action of their resultant.

   (b) A uniform cubical box of length b is placed on the top of a fixed sphere. Find the least radius of the sphere for which the equilibrium is stable.

6. A semicircular disc rests in a vertical plane with its curved edge on a rough horizontal and an equally rough vertical plane, the coefficient of friction being μ. Show that the greatest angle that the bounding dismeter can make with the horizontal plane is

$$\sin^{-1}\left(\frac{3\pi}{4}.\frac{\mu+\mu^3}{1+\mu^3}\right)$$ 7

7. An isosceles triangular lamina with its plane vertical rests downwards between two smooth pegs in the same horizontal line. Show that there will be equilibrium if the base makes an angle $\sin^{-1}\left(\cos^2\alpha\right)$ with the vertical ; 2α being the vertical angle of the lamina, and the length of the base being three times the distance between the pegs. 7

8. A uniform rod of length $2l$ is attached by smooth rings at both ends to a parabolic wire fixed with its axis vertical and vertex downwards, and of latus rectum $4a$. Show that the angle θ which the rod makes with the horizontal in a slanting position of equilibrium is given by $\cos^2\theta = 2a/l$; and that, if these positions exist, they are stable. 7

9. Show that the distance from the cusp of the centroid of the area of the cardiod $r = a\left(1+\cos\theta\right)$, when the density at any point varies as the n-th power of the distance from the cusp, is

$$\frac{(n+2)(2n+5)}{(n+3)(n+4)}\,.\,a\,.$$ 7

10. Three forces act along the straight lines $x=0,\ y-z=a;\ y=0,\ z-x=a;\ z=0,\ x-y=a$. Show that they cannot reduce to a couple. Prove also that if the system reduces to a single force, its line of action must lie in the surface

$x^2+y^2+z^2-2yz-2zx-2xy=a^2$. 7

Group - C

(Marks : 25)

Answer question no. 11 and any *three* from the rest.

11. Answer any *one* question : 4

(a) Find the equation of the momental ellipsoid at the centre of the elliptic plate of major and minor axes $2a$, $2b$ respectively.

(b) Prove that the centre of suspension and centre of oscillation of a compound pendulum are interchangable.

12. Show that at the centre of the quadrant of an ellipse the principal axes in its plane are inclined at an angle

$$\frac{1}{2}\tan^{-1}\left[\frac{4ab}{\pi\left(a^2-b^2\right)}\right].$$ 7

13. A rod of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi-vertical angle α. Show that $\omega^2 = \dfrac{3g}{4a\cos\alpha}$ or $\alpha = \cos^{-1}\left(\dfrac{3g}{4a\omega^2}\right)$. 7

14. Show that the kinetic Energy of a rigid body of mass M w.r.t. the fixed point can be expressed as $\frac{1}{2}MV^2 + \frac{1}{2}MK^2\dot{\theta}^2$.

    (Symbols have their usual meanings). 7

15. A uniform rod is held in a vertical position with one end resting upon a perfectly rough table, and when released rotates about the end in contact with the table. Discuss the motion. 7

16. *AB* and *BC* are two equal similar rods freely hinged at *B* and lie in a straight line on a table. The end *A* is struck by a blow perpendicular to *AB*. Show that the resulting velocity of the end *A* is $3\frac{1}{2}$ times that of the point *B*. 7

Group - D

(Marks : 20)

Answer question no. 17 and any *two* from the rest.

17. Answer any *one* question :

    (a) A closed tube in the form of an equilateral triangle contains equal volumes of three liquids which do not mix and is placed with the lower side horizontal. Prove that if the density of the liquids are in *AP*, their surfaces of separation will be at points of trisection of the sides of the triangle. 6

    (b) A square is placed in a liquid with one side in the surface. Show how to draw a horizontal line, in the surface dividing it into two portions, the thrusts on which are the same. 6

18. A fine circular tube in the vertical plane contains a column of liquid of density ρ, which subtends a right angle at the centre and a column of density σ subtending an angle α. Prove that the radius through the common surface makes with the vertical an angle

$$\tan^{-1}\left[\frac{\rho-\sigma+\sigma\cos\alpha}{\rho+\sigma\sin\alpha}\right].$$ 7

19. If the absolute temperature T diminishes upwards in the atmosphere according to the law $T = T_0(1+\beta z)^{-1}$, where β is a constant. Show that the pressure at a height z is given by $p = p_0 e^{-\left(\frac{z}{H}+\frac{\beta z^2}{2H}\right)}$, where H being the height of the homogeneous atmosphere. 7

20. Prove that is the forces per unit mass at (x, y, z) parallel to the axes are $y(a-z), x(a-z)$ and xy respectively, the surfaces of equal pressure are hyperbolic paraboloid and the curves of equal pressure and density are rectangular hyperbolas. 7