UG 5th Semester Examination 2021

PHYSICS (Honours)

Paper Code : DSE-1

[CBCS]

Full Marks : 25 **Time : Two Hours**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Paper : DSE-1A

Advanced Mathematical Methods-I

1. Answer any ***six*** of the following questions. 6×2=12

(a) Find the residue of $(z^2-2z)/(z+1)^2(z^2+4)$ at the double pole for $z=-1$.

(b) By comparing series expansions, show that

$$\tan^{-1} x = \frac{i}{2}\ln\left(\frac{1-i\,x}{1+i\,x}\right)$$

(c) Determine whether or not each of the following sets is a null set.

(i)$X=\{x: x^2=9, 2x=4\}$

(ii)$Y=\{x: x\neq x\}$

(d) Let the functions $f: \boldsymbol{R}\to\boldsymbol{R}$ and $g: \boldsymbol{R}\to\boldsymbol{R}$ be defined by

$f(x)=2x+1, g(x)=x^2-2$

Find the formula defining product function $g\circ f$ and $f\circ g$.

(e) Given the following three vectors from the vector space of real 2×2 matrices:

$$|1\rangle=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, |2\rangle=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}, |3\rangle=\begin{pmatrix}-2 & -1\\ 0 & -2\end{pmatrix}$$

Determine whether they are linearly dependent or independent.

(f) Given that $A=\begin{bmatrix}-5 & -3\\ 2 & 0\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find the value of A^3.

(g) Let $T_{ij}=\sum_k \epsilon_{ijk}a_k$ and $\beta_k=\sum_{ij}\epsilon_{ijk}T_{ij}$, where ϵ_{ijk} is the Levi-Civita operator, defined to be zero if two of the indices coincide, and +1 and -1, depending on whether ijk is even or odd permutation of 1,2, 3. Find the value of β_3.

(h) Show that an $n\times n$ orthogonal matrix has $n(n-1)/2$ independent parameters.

(i) If A and B are two elements such that $P(A)=\frac{2}{3}$, $P(B)=\frac{3}{4}$, $P(A\cap B)=\frac{1}{2}$, then find $P(A\cup B)$.

2. Answer any ***four*** of the following questions. 4×5=20

(a) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$. 5

(b) If the series of the co-efficient $\sum a_n$ and $\sum b_n$ are absolutely convergent, show that the Fourier series

$$\sum(a_n \cos nx + b_n \sin nx)$$

is uniformly convergent for $-\infty < x < \infty$. 5

(c) Consider the vector space E_3 with the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis

$$|1\rangle = (1,1,1), |2\rangle = (0,1,1), |3\rangle = (0,0,1)$$

into an orthonormal basis. 5

(d) Show that the matrices

$$M(\theta, x, y) = \begin{pmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{pmatrix}$$

Where $0 \le \theta < 2\pi, -\infty < x < \infty, -\infty < y < \infty$, form a group under multiplication. Show that those $M(\theta, x, y)$ for which $\theta = 0$ form a subgroup and identify its cosets. 5

(e) A group $\mathcal{G}$ has four elements I, X, Y and Z, which satisfy $X^2 = Y^2 = Z^2 = XYZ = I$. Show that $\mathcal{G}$ is Abelian and hence deduce the form of its character table.

Show that the matrices

$D(I) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $D(X) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $D(Y) = \begin{pmatrix} -1 & -p \\ 0 & 1 \end{pmatrix}$ and $D(Z) = \begin{pmatrix} 1 & p \\ 0 & -1 \end{pmatrix}$

where p is a real number, form a representation D of $\mathcal{G}$. Find its characters and decompose it into irreducible representations. 5

(f) Verify that the sequence $\delta_n(x)$, based on the function

$$\delta_n(x) = \begin{cases} 0, & x < 0 \\ n\, e^{-nx}, & x > 0 \end{cases}$$

is a delta sequence. Note that the singularity is at $+0$ the positive side of the origin. 5

(g)The elements of the quaternion group, $\mathbb{Q}$, are the set

$$\{1, -1, i, -i, j, -j, k, -k\},$$

With $i^2 = j^2 = k^2 = -1, ij = k$ and its cyclic permutations, and $ji = -k$ and its cyclic permutations. Find the proper subgroups of $\mathbb{Q}$ and the corresponding cosets. 5

Paper : DSE-1B

Nuclear and Particle Physics

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any ***six*** of the following questions. 6×2=12

(a) Explain why electrons cannot be a part of a nucleus.

(b) A nucleus with A = 235 splits into two nuclei of mass numbers in the ratio 2:1. Find the radii of the new nuclei. Given nuclear radius parameter (r_0) = 1.4 fermi

(c) What is pair production? Why cannot it take place in vacuum?

(d) Define Q-value and reaction cross-section of a nuclear reaction.

(e) Complete the nuclear reaction $^{15}N_7$ (p, d), including the compound nucleus in the intermediate stage.

(f) Why a cyclotron cannot be used for accelerating electrons?

(g) A reactor is developing nuclear energy at a rate of 32000 KeV. How many atoms of ^{235}U would undergo fission per second? Assume that on an average 200 MeV of energy is released per fission.

(h) The isospin, baryon number, and strangeness of a particle are given by $I = 0$, $B = +1$ and $S = -3$ respectively. Find the electric charge of the particle.

(i) What are the quark contents of a proton and a π^+?

3. Answer any ***four*** of the following questions. 4×5=20

(a) [i] Deduce the law of successive disintegration in the case of the following radioactive transformation:

$$\boldsymbol{P} \longrightarrow (\lambda_1) \longrightarrow \boldsymbol{Q} \longrightarrow (\lambda_2) \longrightarrow \boldsymbol{R}\ (stable)$$

where λ_1 and λ_2 are the disintegration constants of $\boldsymbol{P}$ and $\boldsymbol{Q}$ respectively. At time $t = 0$, number of P particles is N_0, number of Q and R particles are both zero.

[ii] What are the transient and the secular equilibria in connection with the radioactive disintegration.

[3+2]

(b) Write down the formal expression of binding energy of a nucleus. Show graphically the variation of average binding energy per nucleon with mass number and explain the importance of binding energy.

[2+1+2]

(c) [i] Though a neutron is an uncharged particle, it has a magnetic moment $M_n = -1.913\mu_N$, where μ_N is the nuclear magneton. Why?

[ii] What is the number of leptons? Give their names.

[iii] In case of intrinsic spin of a neutron, give the gyromagnetic ratio.

[2+(1+1)+1]

(d) [i] What is Bohr's independence hypothesis for a compound nuclear reaction?

[ii] What is a resonant reaction? Is it as fast as a direct nuclear reaction?

[iii] Find the Cerenkov radiation angle for an electron moving with a velocity $0.577c$ inside a material of refractive index 2.0.

[2+(1+1)+1]

(e) [i] Obtain the betatron condition in connection with the operation of the machine.

[ii] In a certain betatron, the diameter of the stable orbit is 1.5m. The maximum magnetic field at the orbit is 0.4T, operating at 50 Hz. Calculate the average energy gained per revolution and the final energy of the electrons.

[2+(1+2)]

(f) Describe the construction and working principle of a GM counter. [2+3]

(g) [i] Identify the unknown particle X using conservation laws:

$$\mu^- + p \longrightarrow n + X,$$

[ii] Explain why one cannot find a baryon with strangeness -2 and electric charge +1.

[iii] Identify the type of interaction (strong, weak or electromagnetic) which is responsible for the following decay

$$\pi^0 \longrightarrow \gamma + \gamma$$

[2+2+1]