

UG/1st Sem/G/20 (CBCS)

2020

MATHEMATICS (General)

Paper : MTMG - DC-1/GE-1

[CBCS]

Full Marks : 32

Time : Two Hours

The figures in the margin indicate full marks.

*Candidates are required to give their answers
in their own words as far as practicable.*

Notations and symbols have their usual meanings.

Group - A

1. Answer any **four** questions.

$1 \times 4 = 4$

- (a) For what values of λ , the equation $x^2 + \lambda xy - 2y^2 + 3y - 1 = 0$ represents a pair of straight lines?
- (b) Show that if $a \mid b$ and $a \mid c$, then $a \mid (b + c)$.
- (c) Find the complex conjugate of $\frac{2+3i}{5+7i}$.
- (d) Find an equation of degree 3 with real coefficient having roots 2 and $2 + 3i$.
- (e) Show that the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is orthogonal.
- (f) Find the pole of the line $x + 2y + 3 = 0$ with respect to the circle $x^2 + y^2 - 2x + 5 = 0$.
- (g) Find the equation of tangent(s) at the origin to the curve $2x^2 + 5xy + 3y^2 + 4x - 10y = 0$.

Group - B

Answer any *two* questions.

5×2=10

2. If $u + iv = \operatorname{cosec}(x + iy)$, then show that

$$(u^2 + v^2)^2 = \frac{u^2}{\sin^2 x} - \frac{v^2}{\cos^2 x} = \frac{u^2}{\cosh^2 y} + \frac{v^2}{\sinh^2 y}. \quad [5]$$

3. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha)$. [5]

4. Prove that the feet of the normals from the point (α, β) to the parabola $y^2 = 4ax$ lie on the curve $xy - (\alpha - 2a)y = 2a\beta$. [5]

5. Find the general equation of a conic in polar form (r, θ) , with focus as a pole. [5]

Group - C

Answer any *two* questions.

9×2=18

6. (a) If one of the straight lines given by the equation $ax^2 + 2hxy + by^2 = 0$ coincides with one of the straight lines given by $a'x^2 + 2h'xy + b'y^2 = 0$ and the remaining two straight lines are at right angles, then show that $h\left(\frac{1}{b} - \frac{1}{a}\right) = h'\left(\frac{1}{b'} - \frac{1}{a'}\right)$. [5]

(b) Reduce the equation $x^2 + 4y^2 - 20x + 40y + 196 = 0$ to its canonical form. [4]

7. (a) Find the equations of the planes parallel to the plane $16x + 12y - 15z + 75 = 0$ and at a distance 4 from it. [5]

(b) Find the image of the point $(1, -2, 3)$ in the plane $2x - 3y + 2z + 3 = 0$. [4]

8. (a) Reduce the following matrix to row reduced echelon matrix and also find its rank

$$\begin{pmatrix} 1 & 3 & 4 & 5 \\ 1 & 2 & 5 & 7 \\ 1 & 5 & 0 & 1 \end{pmatrix}. \quad [5]$$

(b) Show that

$$\begin{vmatrix} a - b - c & 2a & 2a \\ 2b & b - c - a & 2b \\ 2c & 2c & c - a - b \end{vmatrix} = (a + b + c)^3. \quad [4]$$