

UG/1st Sem/H/20 (CBCS)

2020

MATHEMATICS (Honours)

Paper : MTMH - DC-01

[CBCS]

Full Marks : 32

Time : Two Hours

The figures in the margin indicate full marks.

*Candidates are required to give their answers
in their own words as far as practicable.*

Notations and symbols have their usual meanings.

Group - A

1. Answer any **four** questions.

1 × 4 = 4

- (a) Prove that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x|x|$ is differentiable at every point.
- (b) Show that $\lim_{x \rightarrow 0} \cos \frac{1}{x}$ does not exist.
- (c) State Leibnitz rule of successive differentiation.
- (d) Obtain an equation of second degree that represents a pair of straight lines passing through the origin.
- (e) How many normals can be drawn from a given point to a parabola?
- (f) Obtain the radius of the circle of intersection of a sphere of radius 13 cm by a plane at a distance 5 cm from the centre of the sphere.
- (g) Write down the name of the quadric surface represented by the equation $2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z - 5 = 0$.

Group - B

Answer any *two* questions.

5×2=10

2. If $\log y = \tan^{-1} x$, then prove that $(1+x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)y_n = 0$. [5]
3. Find the envelope of circles whose centre lie on the rectangular hyperbola $xy = c^2$ and passes through the origin. [5]
4. Prove that the chord of contact of two mutually perpendicular tangents drawn from a point to a parabola passes through its focus. [5]
5. If the guiding curve of a right circular cylinder is the circle $x^2 + y^2 + z^2 = 9, x - y + z = 3$, then find the equation of the cylinder. [5]

Group - C

Answer any *two* questions.

9×2=18

6. (a) Use L'Hospital's rule to evaluate

$$\lim_{x \rightarrow -4} \frac{\sin(\pi x)}{x^2 - 16}. \quad [2]$$

- (b) Use Taylor's theorem to prove that

$$1 + \frac{x}{2} - \frac{x^3}{8} < \sqrt{1+x} < 1 + \frac{x}{2}, \quad \text{if } x > 0. \quad [3]$$

- (c) Obtain the condition for a general equation of second degree that represents a pair of straight lines. [4]

7. (a) Plot the graph of e^x and use it to plot the graph of the function e^{x+5} . [2]
- (b) Find the length of the radius of curvature of the rectangular spiral $r = ae^{\theta \cot \alpha}$ at (r, θ) . [3]
- (c) Find the locus of the point of intersection of the perpendicular generators of the following hyperboloid of one sheet $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$. [4]
8. (a) If $I_n = \int \sin^n x dx$, then show that $I_n = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2}$. [3]
- (b) Find the asymptotes of the curve $(y-2x)^2(y-x) - 3(y-2x)(y-x) + 2(y-x) + 1 = 0$. [4]
- (c) Find the equation of the bisectors of the angles between the pair of straight lines represented by $8x^2 + 10xy + 3y^2 + 26x + 16y + 21 = 0$. [2]
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