P-I(1+1+1) G/21 (N)

KALIACHAK COLLEGE

2021

MATHEMATICS (General)

Paper Code: I-A

[New Syllabus]

Full Marks: 50 Time: One Hour

Notations and symbols have their usual meanings.

Choose the correct answer.
Each question carries 2 marks

1. If ω be an imaginary cube root of unity, then the value of $(1-\omega+\omega^2)(1+\omega-\omega^2)$ is

   A. 1
   B. 2
   C. 3
   D. 4

2. The remainder when $3x^2+4x-11$ is divided by $(x-1)$ is

   A. -4
   B. -6
   C. -8
   D. -10

3. If $A=\{1,2,3\}$ and $B=\{2,3,4\}$, then $(A\times B)-(B\times A)$ is

   A. $\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$
   B. $\{(1,2),(1,3),(1,4),(2,3),(3,4)\}$
   C. $\{(1,2),(1,3),(1,4),(2,4),(4,3)\}$
   D. $\{(1,2),(1,3),(1,4),(4,2),(3,4)\}$

4. If A is a non-singular symmetric matrix, then A^{-1} is

   A. orthogonal
   B. symmetric
   C. skew-symmetric

D. none of the above

5. The vectors whose position vectors are $\mathbf{a}, \mathbf{b}, \mathbf{3a} - \mathbf{2b}$ are

A. collinear

B. non-collinear

C. independent

D. none of the above

6. The vectors $2\hat{i} - 6\hat{j} - 3\hat{k}$ and $3\hat{i} - 2\hat{j} + 6\hat{k}$ are

A. perpendicular

B. non-perpendicular

C. parallel

D. non-parallel

7. The vectors $4\hat{i} + 3\hat{j} - \hat{k}$, $2\hat{i} + \hat{j} - 3\hat{k}$ and $\hat{i} - 4\hat{k}$ are

A. linearly dependent

B. linearly independent

C. mutually perpendicular

D. none of the above

8. The triangle ABC whose vertices are $A(2\vec{i} - \vec{j} + \vec{k}), B(\vec{i} - 3\vec{j} - 5\vec{k}), C(3\vec{i} - 4\vec{j} - 4\vec{k})$ is

A. isosceles

B. equilateral

C. right-angled

D. none of the above

9. The series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ...$ is

A. convergent

B. divergent

C. absolutely convergent

D. none of the above

10. Value of $\lim_{x \to 0} \sqrt{x} sin \frac{1}{x^2}$ is

A. 0

B. 1

C. 2

D. 3

11. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} 0, & \text{if } x \in \mathbb{Q} \\ 1, & \text{if } x \notin \mathbb{Q} \end{cases}$$

A. is continuous everywhere on $\mathbb{R}$

B. is continuous at $x = 0, 1$

C. is discontinuous everywhere on $\mathbb{R}$

D. none of the above

12. The limit $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$

A. has value 0

B. has value 1

C. has value 2

D. does not exist

13. $\lim\limits_{x\to 0+} (\frac{1}{x} - \frac{1}{sinx})$ is

A. 0

B. 1

C. -1

D. 2

14. Angle between the pair of straight lines $4x^2 - 24xy + 11y^2 = 0$ is

A. $tan^{-1}(\frac{3}{4})$

B. $tan^{-1}(\frac{4}{5})$

C. $tan^{-1}(\frac{4}{3})$

D. $tan^{-1}(\frac{5}{4})$

15. Angle between the planes $2x - y + 3z + 7 = 0$ and $x - 2y - 3z + 8 = 0$ is

A. $cos^{-1}(\frac{5}{12})$

B. $cos^{-1}(\frac{5}{14})$

C. $cos^{-1}(-\frac{5}{14})$

D. none of the above

16. Perpendicular distance of the plane $x + 2y - 2z = 9$ from the point $(2, 3, -5)$ is

A. 2

B. 4

C. 1

D. 3

17. The straight lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

are

A. non-coplanar
B. coplanar
C. orthogonal
D. none of the above

18. The mapping $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = cosx$ is

A. injective
B. surjective
C. neither injective nor surjective
D. bijective

19. Which of the following is a subspace of $\mathbb{R}^3$ over $\mathbb{R}$?

A. $S = \{(x, y, z) \in \mathbb{R}^3 : 2x + y + 3z = 0\}$
B. $S = \{(x, y, z) \in \mathbb{R}^3 : x - 2y - z = 2\}$
C. $S = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 1\}$
D. $S = \{(x, y, z) \in \mathbb{R}^3 : 3y + z + 1 = 0\}$

20. The value of k for which the set of vectors

$$S = \{(k, k, 1), (k, 1, k), (1, k, k)\}$$

is linearly independent

A. 1,$\frac{1}{2}$
B. -1,$\frac{1}{2}$
C. -1,$-\frac{1}{2}$
D. 1,$-\frac{1}{2}$

21. Which of the following is not a field

A. ($\mathbb{Z}$,+,.)
B. ($\mathbb{Q}$,+,.)
C. ($\mathbb{Z}_5$,+,.)
D. ($\mathbb{R}$,+,.)

22. Characteristic of the ring $(\mathbb{Z}_3, +, .)$ is

A. 0

B. 1

C. 2

D. 3

23. Order of the alternating group A_5 is

A. 5

B. 60

C. 120

D. none of these

24. Number of generators of the group $(\mathbb{Z}, +)$ is

A. 1

B. 2

C. 3

D. 4

25. For $z = -1 + i$, value of $arg z$ is

A. $\frac{\pi}{4}$

B. $\frac{2\pi}{4}$

C. $\frac{3\pi}{4}$

D. $\frac{5\pi}{4}$

KALIACHAK COLLEGE

2021

MATHEMATICS (General)

Paper Code: I-B

[New Syllabus]

Full Marks: 100 Time: Three Hours

The figures in the margin indicate full marks.
Notations and symbols have their usual meanings.

Group-A
(15 Marks)
Answer any **three** questions

1. If x and y be complex, then prove, from the definitions of sine and cosine of complex numbers, that

$$cosx - cosy = 2sin\tfrac{1}{2}(y+x)sin\tfrac{1}{2}(y-x)$$

2. Show that

$$\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$$

is a perfect square.

3. Let A be a square matrix. Prove that $A + A^T$ is symmetric and $A - A^T$ is skew-symmetric.

4. If $x^3 + 3px + q$ has a factor of the form $(x-\alpha)^2$, then prove that $q^2 + 4p^3 = 0$.

5. If $\alpha, \beta, \gamma, \delta$ be the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, then find the value of $\sum \alpha^2\beta\gamma$.

Group-B
(15 Marks)
Answer any **three** questions.

6. Let $f : A \to B$ be a bijective mapping. Prove that the mapping $f^{-1} : B \to A$ is also a bijection and $(f^{-1})^{-1} = f$.

7. In a group $(G, *)$, prove that $(a * b)^{-1} = b^{-1} * a^{-1}$, for all $a, b \in G$.

8. Prove that if U and W be subspaces of a vector space V over a field F, then the union $U \cup W$ is a subspace of V if either $U \subseteq W$ or $W \subseteq U$.

9. Let V be a real vector space with $\{\alpha, \beta, \gamma\}$ as a basis. Prove that the set $\{\alpha + \beta + \gamma, \beta + \gamma, \gamma\}$ is also a basis of V.

10. Consider the set of all matrices of the form

$$\begin{pmatrix} a & b \\ 2b & a \end{pmatrix}$$

where $a, b \in \mathbb{R}$. Examine if the set forms a field under the matrix addition and multiplication.
is a field.

Group-C
(10 Marks)
Answer any **two** questions.

11. Express $\mathbf{b}$ as a sum of two vectors such that one part is parallel to $\mathbf{a}$ and the other part is perpendicular to $\mathbf{a}$.
12. By vector method, prove that diagonals of a rhombus bisect each other at right angle.
13. Forces $\vec{F_1} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{F_2} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{F_3} = \hat{i} - \hat{j} + \hat{k}$ act on a particle at a point A. Find the work done by these forces when the particle is displaced from the position A to B where $4\hat{i} + \hat{j} - 3\hat{k}$ and $6\hat{i} + 5\hat{j} - 6\hat{k}$ are the position vectors of A and B respectively relative to some fixed point O as origin.
14. Find the vector equation of the plane through the points $(3, 2, 1), (2, 4, 7)$ and perpendicular to the plane $3x + 4y + 2z = 35$.

Group-D
(25 Marks)
Answer any **five** questions.

15. Reduce the following equation to the canonical form and state the type of the conic:

$$8x^2 + 12xy + 13y^2 = 884.$$

16. Show that the line $\frac{l}{r} = Acos\theta + Bsin\theta$ touches the circle $r = 2acos\theta$, if

$$a^2B^2 + 2A\alpha l = l^2.$$

17. $ax + by + c = 0$ bisects an angle between a pair of straight lines of which one is $lx + my + n = 0$. Show that the other line of the pair is

$$(a^2 + b^2)(lx + my + n) - 2(al + bm)(ax + by + c) = 0$$

18. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two lines equidistant from the origin, show that $f^4 - g^4 = c(bf^2 - ag^2)$.
19. Find the equation of the cone whose vertex is $(1, 1, 3)$ and guiding curve is

$$4x^2 + y^2 = 1, z = 4.$$

20. Find the image of the point $(-3, 5, 2)$ in the plane $2x - y + z = 0$.
21. A sphere of radius $2d$ passes through the origin and meets the co-ordinate axes at A, B, C. Show that the locus of the centroid of the tetrahedron with vertices at A, B, C and the origin is $x^2 + y^2 + z^2 = d^2$.

Group-E
(35 Marks)
Answer any **seven** questions.

22. Prove that the sequence $\{u_n\}$ defined by

$$u_1 = \sqrt{3} \text{ and } u_{n+1} = \sqrt{3u_n}, \text{ for } n \geq 1$$

converges to 3.
23. Test the convergence of the series

$$1 + \frac{1}{1!} + \frac{2^2}{2!} + \frac{3^3}{3!} + ...$$

24. Discuss the continuity of the function

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$$

at $x = 0$.
25. Evaluate $\lim_{x \to 0} (\frac{sinx}{x})^{\frac{1}{x}}$.
26. Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters a, b are connected by $ab = c^2$.
27. Find the radius of curvature at any point of the curve

$$r = a(1 - cos\theta).$$

28. Find the pedal equation of the curve

$$x = ae^{\theta}(sin\theta - cos\theta), y = ae^{\theta}(sin\theta + cos\theta).$$

29. If $V = cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, then verify that $cosV = \frac{x+y}{\sqrt{x}+\sqrt{y}}$ is a homogeneous function of x, y of degree $\frac{1}{2}$. Hence prove that

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} + \frac{1}{2}cotV = 0.$$

30. Find the asymptotes of the curve:

$$2x^3 + 3x^2y - 3xy^2 - 2y^3 + 3x^2 - 3y^2 + y - 3 = 0.$$

31. Find the maximum value of $f(x, y, z) = x^2y^2z^2$ subject to the condition

$$x^2 + y^2 + z^2 = c^2.$$